A workshop on atmospheric boundary Layer processes
Howard University Beltsville Campus

Meteorological Boundary Layer Box models

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Goals

A. Learn the basic thermodynamic attributes of the convective atmospheric boundary layer.

B. Investigate the salient processes influencing the thermodynamics of the convective atmospheric boundary layer.

C. Apply the most basic methods to determine the depth of the convective atmospheric boundary layer.

Expected outcomes

1. Participants will identify the key attributes of the convective atmospheric boundary layer.

2. Participants will gain knowledge of the atmospheric processes exerting the greatest influences on the growth of the depth of the convective atmospheric boundary layer.

3. Participants will develop skills to develop numerical models to determine temporal variations in the depth of the convective atmospheric boundary layer.

Readings

Betts, A. (1973). Non-precipitating cumulus convection and its parameterization. Quarterly Journal of the Royal Meteorological Society, 99:178–196.

Driedonks, A. (1982) Sensitivity analysis of the equations for a convective mixed layer. Boundary-Layer Meteorology, 22:475–480.

Lilly D.K. (1968). Models of cloud-topped mixed layers under a strong inversion. Quarterly Journal of the Royal Meteorological Society, 94: 292-309.

Tennekes, H. (1973). A model for the dynamics of the inversion above a convective boundary layer. Journal of the Atmospheric Sciences, 30:558–567.

Theoretical background

Bulk or slab models (Betts 1973, Driedonks 1982, Lilly 1968, and Tennekes 1973) use the vertically integrated or layer averaged forms of the equation of motion and thermodynamic variables to derive the depth (Z_i) of the mixed layer. The evolution of mixed layer depth over heated land surfaces occurs in several stages which include (1) the erosion of the nocturnal inversion and formation of a shallow mixed layer, which slowly deepens in early morning hours after sunrise; (2) the rapid growth of the mixed layer during the mid-morning period; (3) the slow growth of the deep mixed layer in the afternoon period; and (4) the decay of turbulence in the mixed layer of nearly constant depth during the late afternoon and evening transition period.

From the conservation of energy, the growth rate of the mixed layer, Z_i, is related to the rate of warming of the mixed layer ($\frac{\partial \theta_{Vm}}{\partial t}$) and can be expressed as

$$\frac{\partial Z_i}{\partial t} = \frac{1}{\gamma_{\theta_V}} \frac{\partial \theta_{Vm}}{\partial t} \tag{1}$$

Where $\gamma_{\theta_V} = \frac{\partial \theta_V}{\partial z}$ is the temperature gradient above the mixed layer height, θ_V is the virtual potential temperature, and θ_{Vm} is the mixed layer virtual potential temperature. The γ_{θ_V} represents the virtual potential temperature gradient above the top of the mixed layer. Also, from the conservation of energy (see Equation 4 in Lilly 1968), one can determine that the rate of change of mean virtual potential temperature of the mixed layer (θ_{Vm}) can be expressed as:

$$\frac{\partial \theta_{Vm}}{\partial t} = \frac{1}{Z_i} \left[\left(\overline{w'\theta'_V}\right)_{sfc} - \left(\overline{w'\theta'_V}\right)_{Z_i} \right] \tag{2}$$

where $\left(\overline{w'\theta'_V}\right)_{sfc}$ and $\left(\overline{w'\theta'_V}\right)_{Z_i}$ represent the kinematic virtual heat fluxes at the surface and at the top of the mixed layer, respectively. The rate of change of mixed layer virtual potential temperature ($\frac{\partial \theta_{Vm}}{\partial t}$) can be substituted in equation (1) to obtain an expression for the growth rate of the mixed layer height which is given as

$$\frac{\partial Z_i}{\partial t} = \frac{1}{Z_i \, \gamma_{\theta_V}} \left[\left(\overline{w'\theta'_V}\right)_{sfc} - \left(\overline{w'\theta'_V}\right)_{Z_i} \right] \tag{3}$$

The kinematic virtual heat flux at the top of the boundary layer (entrainment flux) can be expressed as a function of the surface kinematic virtual heat flux as expressed below.

$$\left(\overline{w'\theta'_V}\right)_{Z_i} = -\, C \left(\overline{w'\theta'_V}\right)_{sfc} \tag{4}$$

By combining relationships (1) to (3) one can obtain a relationship for the change in the depth of the mixed layer (∂Z_i).

$$\partial Z_i = \frac{1 + C}{Z_i \, \gamma_{\theta_V}} \left[\left(\overline{w'\theta'_V}\right)_{sfc} \right] \partial t \tag{5}$$

Application of relation (5) assumes that Z_i applies to initial mixed layer depth or its value at the previous time step. The value of C, which is the closure parameter, can vary depending on the site characteristics. In the case where there is no energy entrainment, the value of C becomes 0. The value of $\gamma_{\theta V}$ can be estimated from the initial upper air sounding data.

Also, it is possible to perform a numerical integration of equation (3) to obtain an analytical expression to estimate the mixed layer depth. The integral of the kinematic surface heat flux can be evaluated if it is assumed constant over the time interval from t_o to t as

$$Z_i(t) = \left[Z_i^2(t_0) + \frac{2(1+C)}{\gamma_{\theta_V}} \left(\overline{w'\theta'_V}\right)_{sfc} (t - t_0) \right]^{\frac{1}{2}} \tag{6}$$

In the early morning hours (i.e., around sunrise), the initial depth of the mixed layer is ordinarily 0 m (i.e., $Z_i(t_0) = 0$ m). For such conditions, the depth of the mixed layer can be calculated using equation 7.

$$Z_i(t) = \left[\frac{2(1+C)}{\gamma_{\theta_V}} \left(\overline{w'\theta'_V}\right)_{sfc} (t - t_0)\right]^{\frac{1}{2}} \quad (7)$$

The theory described above is needed to answer the questions provided below.

Available data

Surface energy balance flux data (**BrazilDOY401999 flux data.xls**) are provided for a tropical site located in southwestern Brazil. The data were obtained on 9 February 1999 (Day of Year = DOY 40) at a deforested region in Brazil and corresponded to the rainy season. The following variables are provided.

DOY	Start Hour	End Hour	Rnet	G	H	LE

Names of variables are: H is the sensible heat flux, LE is the latent heat flux, R_{Net} is the net radiation, and G is the soil heat fluxes. All the energy fluxes are given in Watts per m^2.

Data analyses and interpretation

For the conditions associated with the energy fluxes, several temperature changes with altitude will be considered to study the influence of the strength of the temperature gradients on boundary layer growth rates. In addition, several values for the closure parameter, C, will be evaluated to learn how heat entrainment can modify the growth rates of the mixed boundary layer.

A. For each half hour, calculate the virtual heat flux (H_V). The H_V is determined as $H_V = H + 0.07$ LE. To obtain the kinematic virtual heat flux ($(\overline{w'\theta'_V})_{sfc}$), the following relationship needs to be applied: ($(\overline{w'\theta'_V})_{sfc}$) = $H_V/\rho Cp$. The ρ is the air density (=1.2 kg per m^3), Cp is the specific heat capacity of air at constant pressure (= 1004 J per (kg K)), and L_V is the latent heat of vaporization (= 2500 Joules per g). Create two plots on a single page to report the relationship between H versus H_v and the time series of both H and H_V. Figure axes need to have proper titles and units. Identify the times of day when the different between H and H_v are the greatest and provide an explanation for the differences in the magnitude of H and H_V.

B. On a single figure, plot the diel cycle of surface energy balance components (i.e., R_{Net}, H, LE, H, and G). Figure axes need to have proper titles and units. A legend is needed to denote each energy balance component. Provide an explanation for the erratic changes in the energy fluxes during 10 and 16 hours.

Numerical modeling and interpretation

For the conditions associated with the energy fluxes, several temperature changes with altitude will be considered to study the influence of the strength of temperature gradients on boundary layer growth rates. In addition, several values for the closure parameter, C, will be evaluated to learn how heat entrainment can modify the growth rates of the mixed boundary layer.

C. Calculate the height of the mixed layer (Z_i) using equation (6) from 08:00 to 17:30 hours. Use a time step of 0.5 hours to calculate the mixed layer depth. For each half hour, assume that $(\overline{w'\theta'_V})_{sfc}$ remains constant. Determine the temporal variation of Z_i for the following scenarios, assuming that the initial boundary layer depth, $Z_i(t_0)$, is 100 m at 7:30 hours.

   (i) The virtual potential temperature change with height ($\gamma_{\theta V}$) is 4.0 K per km, and the value of C is 0.15.

   (ii) The virtual potential temperature change with height ($\gamma_{\theta V}$) is 8.0 K per km, and the value of C is 0.15.

(iii) The virtual potential temperature change with height ($\gamma_{\theta V}$) is 4.0 K per km, and the value of C is 0.30.

(iv) The virtual potential temperature change with height ($\gamma_{\theta V}$) is 8.0 K per km, and the value of C is 0.30.

For the results obtained above, plot the diurnal patterns of the mixed layer depth on a single graph. For each scenario, explain how the temperature gradients above the mixed layer influences the growth rates of the convective boundary layer. In addition, for each scenario, explain the influence of heat entrainment on the growth rates of the mixed layer.